Monte Carlo simulation of the effect of fiber characteristics on creep life of a fiber tow

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Fatigue of reinforced ceramics at elevated temperatures was numerically evaluated with a fiber dominated, power-law creep model. A Monte Carlo simulation of fiber creep in a uniaxially loaded tow was used to examine the influence of fiber radius, elastic modulus, and strength on creep respose. The simulation permitted variation of both the average magnitude and dispersion of fiber characteristics while maintaining constant power-law creep parameters. A linear increase in creep life was predicted for an increase in mean fiber radius, and a linear decrease in creep life was predicted for an increase in the standard deviation of fiber radii. A linear increase in creep life was predicted for both an increase in mean fiber elastic modulus and standard deviation of elastic moduli. Characteristic fiber strength and Weibull modulus were predicted to have a significant effect on creep life of a SiC fiber tow. An increase in either the characteristic strength or Weibull modulus was predicted to result in an increase in creep life.

1. Background

The mechanical behavior of SiC fibers has been extensively studied [1–36]. These research efforts have focused on understanding the potential of SiC fibers to withstand structural loads as well as extreme temperatures (>500 °C) without active cooling. The creep behavior of SiC fibers is typically modeled by an empirical power-law relationship:

$$\dot{\varepsilon} = C\sigma^n \quad (1)$$

where: $\dot{\varepsilon}$ = creep strain rate; C = constant; σ = applied stress; n = power-law creep exponent.

The majority of the modeling efforts have used literature data [6, 18, 35]. Although this data has been used to gain a better insight into failure mechanisms, it does not provide a comprehensive examination of the all parameters necessary to apply Equation 1, particularly in relation to the variability of fiber characteristics and creep parameters.

A numerical simulation of the creep response of a SiC fiber tow has been reported [22]. While the creep response of the fiber tow was modeled with a power-law relationship, the simulation was based on a Monte Carlo generation of individual fiber characteristics. The fiber characteristics included in the numerical simulation were the fiber radius, elastic modulus, and strength. A limited review of SiC fiber data reported in the literature with regards to these fiber characteristics is given in Table I. Table I shows that the fiber characteristics are known to exhibit variability. The influence of both the magnitude and variability of the fiber radius, elastic modulus, and strength on creep response is currently unknown. Control of these three fiber characteristics is not possible in experimental studies without changing the fiber chemistry. A change in fiber chemistry also changes the creep response of the fibers. However, these three fiber characteristics can be readily isolated in numerical simulations. This communication reports the numerical simulation of the effect of fiber radius, elastic modulus, and strength on the creep response of a SiC fiber tow.

2. Numerical approach

The numerical simulation utilized in the present study modeled an array of fibers aligned parallel to the creep load and perpendicular to a matrix crack [22]. It was assumed that the majority of the creep response occurred in a manner that could be described by a power-law creep model (Equation 1). It was further assumed that individual fibers could be considered uniformly loaded cylinders. This assumption was consistent with small crack opening displacements of matrix cracks. In addition, it was assumed that the crack opening displacement was constant across the fiber array, and was consistent with the expected behavior of a fully cracked composite.

The simulation was based on an isostrain model such that the fiber tow creeps as a unit, but the force on individual fibers was a function of fiber properties. In this manner, the stiffest fibers preferentially carry the

TABLE I Properties for SiC fibers

Properties for SiC Fibers	[7]	[26]	[25]	[4]	[28]	[19] SCS-6	[17] SCS-6
Modulus (E_f) GPa	145	200	200	200	200	350	415
Modulus Deviation	60						
Shear Friction (τ) MPa		2	2		3.6	4.5	1.87, 3.16, 0.87, 2.68, 1.18
Fiber Radius (R) μm	6.9		8				7.1
Radius Deviation	1.3				8.0		
Fiber Strength (S_f) GPa	1.1						
Elastic Limit (s0) MPa			265				
Wiebull Modulus (m)	3.6						
Fiber Volume	0.4	0.5	0.5				
	[5] Nicalon	[5] HI-Nicalon	[12] HP SiC/Si_3N_4	[10] Avco SiC	[24]	[31] Nicalon	[1] HI-Nicalon
Modulus (E_f) GPa	193	269		391	200		
Modulus Deviation				±5			
Shear Friction (τ) MPa					2	10	
Fiber Radius (R) μm	6-9	6-9			8		0.3
Radius Deviation							
Fiber Strength (S_f) GPa	2.96	2.80		4.3 - 5.5	1		
Elastic Limit (s_0) MPa							
Plastic Stress (σ_p) MPa							435 ± 35 composite
Wiebull Modulus (m)							
Fiber Volume			0.30			0.2	0.42
	[36] Nicalon NLM-102	[6] uncoated	[6] coated	[18]	[29] Nicalon	[15]	
Modulus (E_f) GPa	243.3	27,21.9 ($3D$)	35, 22.3 ($3D$)	116.7		140	
Modulus Deviation	40	0.7	1.65	6.1			
Shear Friction (τ) MPa					10		
Fiber Radius (R) μm	11.7-13.8					6	
Radius Deviation							
Fiber Strength (S_f) GPa	2.56	1.61	1.38				
Fiber Strength Deviation	0.41	0.7	6.7				
Wiebull Modulus (m)				3.8	5		
Elastic Limit (s0) MPa				71.3 ± 1.1		70	
Plastic Stress (σ_p) MPa							
Fiber Volume		0.36	0.36	0.4	0.4	0.4	

highest proportion of the applied force. A steady-state creep strain was applied to the material using a general power-law. The physical effect of creep was modeled using an assumption of constant volume to result in a reduction in fiber cross-sectional area. Individual fibers failed when the fiber stress exceeded the fiber strength. As fibers failed, creep load was redistributed to near-neighbor fibers. The creep life was reached when all fibers had failed.

The numerical simulation resulted in two distinct regions of creep behavior [22]. Initially, fiber failure progressed gradually. The region of gradual fiber failure was related to accumulation of creep strain resulting in the failure of low strength or highly loaded fibers. After 10% to 20% of the fibers had failed, the predicted behavior changed to very rapid progression of failure. The region of rapid progression of failure was related to pure mechanical events. It appeared that failure of 10% to 20% of the fibers in the tow resulted in overloading of the majority of the remaining fibers. The result was a cascading series of events that resulted in complete failure without the need of much additional accumulated creep strain. The time spent in the region of rapid fiber failure was only a few percent of the predicted life, and the majority of the creep life was spent in the region of gradual failure.

3. Results

It has been reported that fibers aligned with the principal stress direction dictates the properties of a reinforced ceramic [4, 25]. This observation implies that mechanical performance of fiber-reinforced ceramics can be improved by careful consideration of fiber properties and fiber architecture. The question remains as to what impact fiber radius, elastic modulus, strength, and their variability has on creep life of a reinforced ceramic. The creep response of a SiC tow containing 492 fibers was predicted using the fiber characteristics given in Table II and the creep parameters given in Table III [35]. The radius and elastic modulus of individual fibers were modeled using a Gaussian distribution. The strength of individual fibers was modeled with a Weibull distribution. The average creep life was 15236 creep steps for 269 independent solutions of the simulation.

The influence of the magnitude and variability of the fiber radius, elastic modulus, and strength on predicted creep life is shown in Fig. 1. The data points in Fig. 1

TABLE II Fiber characteristics examined in the present study

Fiber Characteristic	Value	Range Examined
Mean Fiber Radius [7]	6.4 mm	±20% of value
Stand. Dev. Of Fiber Radii [7]	1.2 mm	±20% of value
Characteristic Fiber Strength [36]	482 MPa	±20% of value
Weibull Mod. of Fiber Strength [36]	5	±20% of value
Mean Fiber Elastic Modulus [36]	200 GPa	±20% of value
Stand. Dev. Of Fiber Elastic Moduli [36]	35 GPa	±20% of value

TABLE III Creep parameters [22, 35] used in the present study

Creep Parameter	Value
Fiber Volume Fraction	0.40
Nominal Applied Stress	100 MPa
Stress Elastic Limit	70 MPa
Elastic Strain	1.13%
Creep Strain Rate	3.28%
Creep Power-Law Exponent	3

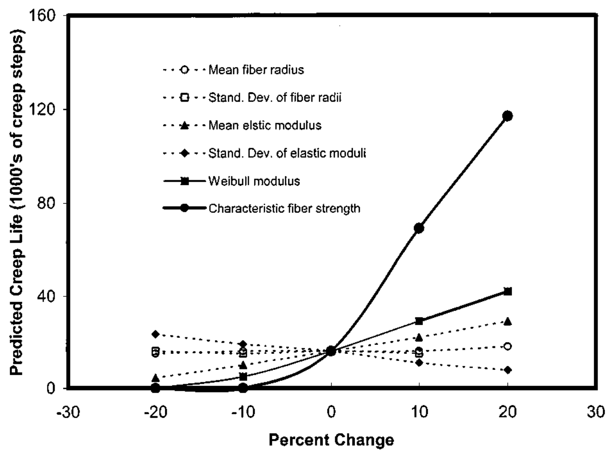

Figure 1 Predicted influence of fiber radius, elastic modulus, and strength on the creep life of a SiC fiber tow with the creep parameters held constant.

are the averages of 269 independent solutions of the simulation. The large number of independent simulations was used to reduce the predicted error, i.e. standard error of the mean, of the average creep life to very small values. Using this approach, the width of the error bars associated with one standard error of the mean was typically on the order of 1% of the magnitude of the average value, and smaller than the height of the data points in Fig. 1.

Increasing the mean fiber radius by 20% resulted in a 0.07% decrease in predicted creep life. However, increasing the mean fiber radius by 10% resulted in a predicted 4.8% increase in predicted creep life. Decreasing the mean fiber radius by 20% resulted in a 4% reduction in predicted creep life. Over the range of mean fiber radii examined in this study, the predicted creep life increased nonlinearly with increased mean fiber radius, but change in the predicted creep life was relatively small.

A 20% reduction in the standard deviation of the radius resulted in a 9% increase in predicted creep life. A 5% decrease in creep life was predicted for a 20% increase in the standard deviation of fiber radius. Over the range of mean fiber radii examined in this study, the creep life was predicted to increase nonlinearly with a decrease in the standard deviation of fiber radii, but the changes were relatively small.

Increasing the mean fiber elastic modulus by 20% resulted in a 93% increase in predicted creep life. Decreasing the mean fiber elastic modulus by 20% resulted in a 68% reduction in predicted creep life. Over the range of mean fiber elastic modulus examined in this study, the predicted creep life increased linearly with mean modulus.

A 20% reduction in the standard deviation of the fiber elastic modulus resulted in a 42% increase in predicted creep life. A 20% increase in the variability of the fiber elastic modulus resulted in a 71% reduction in the predicted creep life. The creep life decreased in a linear manner with increased variability of the fiber elastic modulus. This trend is reasonable since the increase in variation of stiffness results in fewer fibers carrying a disproportionate share of the creep load, and such highly loaded fiber would be the ones expected to fail quickly.

Evident in Fig. 1 is the reversed effects of mean fiber elastic modulus and variability of fiber elastic modulus. As a result of this trend, a simultaneous increase in both mean modulus and variability could result in no net change in predicted creep life. Therefore, improving creep life by increasing the mean fiber elastic modulus would only be effective if there was no increase in variability of fiber modulus.

It is evident in Fig. 1 that changes in predicted creep life caused changes in either fiber radius or modulus are insignificant compared to changes in fiber strength. A 10% reduction of the characteristic fiber strength resulted in a dramatic reduction in predicted creep life. In addition, a 20% reduction in either fiber strength or Weibull modulus resulted in very short predicted creep life. This dramatic reduction in creep life was related to the increase in the number of low strength fibers with a reduction of either the fiber strength or the Weibull modulus. The increase in the number of low strength fibers resulted in failure of a significant number of fibers on initial application of creep loads in the absence of any accumulated creep strain [22]. A linear increase in predicted creep life was observed for an increase in Weibull modulus. The most dramatic effect evident in Fig. 1 is the predicted 443% and 763% increase in creep life resulting from a 10% and 20% increase in characteristic fiber strength, respectively. The two fiber characteristics related to strength (mean strength and Weibull modulus) appear to have the most significant impact on predicted creep life.

4. Conclusions

The influence of three fiber characteristics on the creep life of a SiC fiber tow was predicted using a numerical simulation. Creep was modeled by a power-law creep rate relationship using published values for the power-law parameters. Creep loads were supported by individual fibers in an amount proportional to the elastic modulus of the fiber. In this manner, the strain in each fiber was held constant, and the stiffest fibers carried the largest fraction of the creep load. The simulation examined the effects of both magnitude and dispersion of fiber radii, elastic moduli, and strength on a fiber tow creeping at a constant rate. A decrease in creep life was predicted for an increase in mean fiber radius, and the relationship was nonlinear. A decrease in creep life was predicted for an increase in the standard deviation of fiber radii, and the relationship was nonlinear. A linear relationship was observed for the increase in creep life predicted for an increase in mean fiber elastic modulus and an increase in the standard deviation of elastic moduli. A decrease in creep life was predicted for a decrease in the Weibull modulus of the fiber strengths, and very short creep life was predicted for a 20% reduction in the Weibull modulus. An increase in the Weibull modulus of fiber strengths resulted in a significant increase in predicted creep life. Both 10% and 20% reductions in the characteristic fiber strength resulted in a very short predicted creep life. A dramatic increase in creep life was predicted for an increase in the characteristic fiber strength. The two fiber characteristics related to strength were predicted to have the most significant effect on creep life.

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